Determination of Dielectric Constant and Loss Factor by Free Wave Method. I. Theoretical Part

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Introduction

Recently microwave technique has been developed rapidly, and the determination of dielectric constant and loss factor in microwave region has become important. Great variety of methods has been deviced for this purpose among which the wave guide method is used generally. However earlier experiments were made by means of free wave method. The reason why the free wave method is not used generally lies partly in the incompleteness of the method of calculation to determine dielectric constant and loss factor from the values of reflection or transmission coefficient experimentally obtained.

To find a precise and convenient method for this purpose, it seems desirable to determine theoretically the reflection and transmission coefficient of electromagnetic waves by multiple layer.

In this paper a theoretical formula will be derived for the reflection and transmission coefficient by multiple layer by solving Maxwell's equation in an orthodox way.

At the same time a convenient method will be reported for the determination of dielectric constant and loss factor from the values of reflection or transmission coefficient obtained by experiment.

Reflection and Transmission of Electromagnetic Wave by Multiple Layer. Behavior of Electromagnetic Wave in the Layer.

1. Fundamental Equations and Solutions. —If an electromagnetic wave is propagated in an isotropic and uniform medium, the electric field and magnetic field vectors will satisfy the following Maxwell's equations,[2]

$$\frac{\varepsilon'}{c}\frac{\partial\mathfrak{E}}{\partial t}+\frac{4\pi\sigma}{c}\mathfrak{E}=\mathrm{rot}\,\mathfrak{H} \qquad (1a)$$

$$-\frac{1}{c}\frac{\partial\mathfrak{H}}{\partial t}=\mathrm{rot}\,\mathfrak{E} \qquad (1b)$$

$$\mathrm{div}\,\mathfrak{E}=0 \qquad (1c)$$

$$\mathrm{div}\,\mathfrak{H}=0 \qquad (1d)$$

where ε' denotes dielectric constant, σ conductivity, c the velocity of light and $\mathfrak{E}$ and $\mathfrak{H}$ are electric and magnetic field vectors, respectively.

The electromagnetic wave is assumed to be propagated along x-axis (from minus to plus) and to be linearly polarized. Electric and magnetic field vectors are assumed to lie on y-axis and z-axis, respectively. Then the solution of Eq. (1) is expressed as:[3]

$$\mathfrak{E}_y=ae^{i\omega\left(t-\frac{px}{c}\right)} \qquad (2a)$$

$$\mathfrak{H}_z=ape^{i\omega\left(t-\frac{px}{c}\right)} \qquad (2b)$$

where

$$p^2=\varepsilon'-i\frac{4\pi\sigma}{\omega}. \qquad (3)$$

Here ω denotes circular frequency and a, a constant.

We rewrite Eq. (3) as follows:

$$p^2=\varepsilon'-i\varepsilon'' \qquad (3')$$

where

$$\varepsilon''=\frac{4\pi\sigma}{\omega}. \qquad (4)$$

If we write

$$p=n-ik \qquad (5)$$

then n and k are expressed as

$$n^2=\frac{1}{2}\{\sqrt{\varepsilon'^2+\varepsilon''^2}+\varepsilon'\} \qquad (6a)$$

$$k^2=\frac{1}{2}\{\sqrt{\varepsilon'^2+\varepsilon''^2}-\varepsilon'\}. \qquad (6b)$$

As stated above the electromagnetic wave is assumed to be propagated from $x=-\infty$ to $x=$

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(2) The magnetic permeability of the medium μ is assumed to be 1.

(3) R. Becker "Theorie der Elektrizität" I, Leipzig and Berlin, 1933, p. 186.

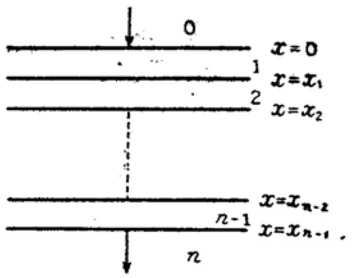

Fig. 1.

$+\infty$ A multiple layer consisting of $(n-1)$ sheets is placed with their boundary surfaces on $y-z$ planes as shown in Fig. 1. The origin of x-coordinate lies on the first boundary surface and the x-coordinate of the boundary surface between r th and $(r-1)$ th layers is noted x_r. Between $x=-\infty$ and $x=0$, there exists a semi-infinite medium, and another between $x=x_{n-1}$ and $x=+\infty$. They are numbered as 0 and n, respectively.

The electric and magnetic field vectors in $0, 1, \ldots, r, \ldots, (n-1), n$ media will be expressed by the following equations.

$$\mathfrak{E}_y^{(0)}=a\{e^{i\omega\left(t-\frac{p_0x}{c}\right)}+\mathfrak{R}_0 e^{i\omega\left(t+\frac{p_0x}{c}\right)}\} \quad (7a)$$

$$\mathfrak{H}_z^{(0)}=ap_0\{e^{i\omega\left(t-\frac{p_0x}{c}\right)}-\mathfrak{R}_0 e^{i\omega\left(t+\frac{p_0x}{c}\right)}\} \quad (7b)$$

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$$\mathfrak{E}_y^{(r)}=a[\vartheta_r e^{i\omega\left\{t-p_r\frac{(x-x_r)}{c}\right\}} + \mathfrak{R}_r e^{i\omega\left\{t+p_r\frac{(x-x_r)}{c}\right\}}] \quad (8a)$$

$$\mathfrak{H}_z^{(r)}=ap_r[\vartheta_r e^{i\omega\left\{t-p_r\frac{(x-x_r)}{c}\right\}} - \mathfrak{R}_r e^{i\omega\left\{t+p_r\frac{(x-x_r)}{c}\right\}}] \quad (8b)$$

$r=1, 2, \ldots, r, \ldots, (n-2), (n-1).$

$$\mathfrak{E}_y^{(n)}=a\vartheta_n e^{i\omega\left\{t-p_n\frac{(x-x_{n-1})}{c}\right\}} \quad (9a)$$

$$\mathfrak{H}_z^{(n)}=ap_r\vartheta_n e^{i\omega\left\{t-p_n\frac{(x-x_{n-1})}{c}\right\}} \quad (9b)$$

$a\mathfrak{R}_r$ and $a\vartheta_r$ denote the complex amplitude of the elecric vector of reflected and penetrating waves at $x=x_r$, respectively.

2. Boundary Conditions.—The boundary conditions require that the electric and magnetic field vectors are continuous on the $x=x_r$ surfaces.

From the continuity of $\mathfrak{E}_y$ on the surface $=0$, we have

$$1+\mathfrak{R}_0=\vartheta_1 e^{i\omega p_1\frac{x_1}{c}}+\mathfrak{R}_1 e^{-i\omega p_1\frac{x_1}{c}} \quad (10_1)$$

and from the continuity of $\mathfrak{H}_z$ on the surface $x=0$, we have

$$p_0(1-\mathfrak{R}_0)=p_1\{\vartheta_1 e^{\frac{i\omega p_1x_1}{c}}-\mathfrak{R}_1 e^{\frac{-i\omega p_1x_1}{c}}\}. \quad (10_2)$$

Similarly on the surface $x=x_1$, the bonndary conditions require

$$\vartheta_1+\mathfrak{R}_1=\vartheta_2 e^{i\omega p_2\frac{(x_2-x_1)}{c}}+\mathfrak{R}_2 e^{-i\omega p_2\frac{(x_2-x_1)}{c}} \quad (10_3)$$

$$p_1(\vartheta_1-\mathfrak{R}_1)=p_2\{\vartheta_2 e^{i\omega p_2\frac{(x_2-x_1)}{c}} - \mathfrak{R}_2 e^{-i\omega p_2\frac{(x_2-x_1)}{c}}\}. \quad (10_4)$$

In general, on the surface $x=x_r$, we have

$$\vartheta_r+\mathfrak{R}_r=\vartheta_{r+1} e^{i\omega p_{r+1}\frac{(x_{r+1}-x_r)}{c}} + \mathfrak{R}_{r+1} e^{-i\omega p_{r+1}\frac{(x_{r+1}-x_r)}{c}} \quad (10_{2r+1})$$

$$p_r(\vartheta_r-\mathfrak{R}_r)=p_{r+1}\{\vartheta_{r+1} e^{i\omega p_{r+1}\frac{(x_{r+1}-x_r)}{c}} - \mathfrak{R}_{r+1} e^{-i\omega p_{r+1}\frac{(x_{r+1}-x_r)}{c}}\} \quad (10_{2r+2})$$

Finally from the boundary condition at $x=x_{n-1}$, the following equations are obtained.

$$\vartheta_{n-1}+\mathfrak{R}_{n-1}=\vartheta_n \quad (10_{2n-1})$$

$$p_{n-1}(\vartheta_{n-1}-\mathfrak{R}_{n-1})=p_n\vartheta_n \quad (10_{2n})$$

We put $i\omega p_{r+1}(x_{r+1}-x_r)/c=a_{r+1}$, and rewrite above equations as follows:

$$\left.\begin{array}{l}
\mathfrak{R}_0-\vartheta_1 e^{+a_1}-\mathfrak{R}_1 e^{-a_1}=-1 \\
-p_0\mathfrak{R}_0-p_1\vartheta_1 e^{+a_1}+p_1\mathfrak{R}_1 e^{-a_1}=-p_0 \\
\vartheta_1+\mathfrak{R}_1-\vartheta_2 e^{+a_2}-\mathfrak{R}_2 e^{-a_2}=0 \\
p_1\vartheta_1-p_1\mathfrak{R}_1-p_2\vartheta_2 e^{+a_2}+p_2\mathfrak{R}_2 e^{-a_2}=0 \\
\cdots\cdots\cdots \\
\cdots\cdots\cdots \\
\vartheta_{n-1}+\mathfrak{R}_{n-1}-\vartheta_n=0 \\
p_{n-1}\vartheta_{n-1}-p_{n-1}\mathfrak{R}_{n-1}-p_n\vartheta_n=0.
\end{array}\right\} \quad (11)$$

We regard the equations (11) as a system of linear simultaneous equations of $2n$ unknown variables $\mathfrak{R}_0, \vartheta_1, \mathfrak{R}_1, \ldots, \vartheta_{n+1}, \mathfrak{R}_{n+1}$, and ϑ_n. For the existence of non-trivial solution of Eqs. (11) it is necessary that the following determinant does not vanish:

$$D = \begin{vmatrix} 1 & -e^{+a_1} & -e^{-a_1} & 0 & 0 & 0 & \cdots & & \\ -p_0 & -p_1e^{+a_1} & p_1e^{-a_1} & 0 & 0 & 0 & \cdots & & \\ 0 & 1 & 1 & -e^{+a_2} & -e^{-a_2} & 0 & \cdots & & \\ 0 & p_1 & -p_1 & -p_2e^{+a_2} & p_2e^{-a_2} & 0 & \cdots & & \\ \cdots & & & & & & & & \\ \cdots & & & & & & & & \\ \cdots & & & & & & & & \\ \cdots & & & & & & 1 & 1 & -1 \\ \cdots & & & & & & p_{n-1} & -p_{n-1} & -p_n \end{vmatrix} \quad (12)$$

We denote the determinant whose mth column is replaced by $(-1, -p_0, 0, \ldots, 0)$, D_m. $\mathfrak{R}_r$ and ϑ_r are expressed as follows:

$$\mathfrak{R}_r = \frac{D_{2r+1}}{D} \quad (13)$$

$$\vartheta_r = \frac{D_{2r}}{D}. \quad (14)$$

From Eqs. (7) (8) (9) and (14), the behavior of electromagnetic wave in the media is completely determined. (Except the arbitrary constant a.)

We may define $\mathfrak{R}_0$ and ϑ_n as complex reflection coefficient and complex transmission coefficient by multiple layer, respectively. These quantities are expressed as follows:

$$\mathfrak{R}_0 = \frac{D_1}{D} \quad (15)$$

$$\vartheta_n = \frac{D_{2n}}{D} \quad (15')$$

For a single layer we have

$$\mathfrak{R}_0 = \frac{D_1}{D}, \quad (16)$$

where

$$D = \begin{vmatrix} 1 & -e^{a} & -e^{-a} & 0 \\ -p_0 & -p_1e^{a} & p_1e^{-a} & 0 \\ 0 & 1 & 1 & -1 \\ 0 & p_1 & -p_1 & -p_2 \end{vmatrix} \quad (17)$$

$$D_1 = \begin{vmatrix} -1 & -e^{a} & -e^{-a} & 0 \\ -p_0 & -p_1e^{a} & p_1e^{-a} & 0 \\ 0 & 1 & 1 & -1 \\ 0 & p_1 & -p_1 & -p_2 \end{vmatrix} \quad (18)$$

$$a = \frac{i\omega p_1 d}{c} \quad (19)$$

and d denotes the thickness of layer 1.

We define $\mathfrak{R}_{01}$ and $\mathfrak{R}_{12}$ by the following equations:

$$\frac{p_1-p_0}{p_1+p_0} = -\mathfrak{R}_{01} \quad (20)$$

$$\frac{p_2-p_1}{p_2+p_1} = -\mathfrak{R}_{12}. \quad (21)$$

$\mathfrak{R}_{01}$ is the complex reflection coefficient of semi-infinite medium 1, with respect to semi-infinite medium 0, when the linearly polarized plane wave penetrate from medium 0 into medium 1 at normal incidence. $\mathfrak{R}_{12}$ is the corresponding quantity with respect to media 1 and 2. Then D_1 and D are expressed as follows:

$$D_1 = e^{+a}(p_0+p_1)(p_1+p_2)\{\mathfrak{R}_{01}+\mathfrak{R}_{12}e^{-2a}\}$$

$$D = e^{+a}(p_0+p_1)(p_1+p_2)\{1+\mathfrak{R}_{01}\mathfrak{R}_{12}e^{-2a}\}$$

and from Eq. (15) we have

$$\mathfrak{R}_0 = \frac{\mathfrak{R}_{01}+\mathfrak{R}_{12}e^{\frac{-2i\omega p_1 d}{c}}}{1+\mathfrak{R}_{01}\mathfrak{R}_{12}e^{\frac{-2i\omega p_1 d}{c}}}. \quad (22)$$

Similarly,

$$\vartheta_1 = \vartheta_{01}e^{\frac{-i\omega p_1 d}{c}} / (1+\mathfrak{R}_{01}\mathfrak{R}_{12}e^{\frac{-2i\omega p_1 d}{c}}) \quad (23)$$

$$\mathfrak{R}_1 = \vartheta_{01}\mathfrak{R}_{12}e^{\frac{-i\omega p_1 d}{c}} / (1+\mathfrak{R}_{01}\mathfrak{R}_{12}e^{\frac{-2i\omega p_1 d}{c}}) \quad (24)$$

$$\vartheta_2 = \vartheta_{01}\vartheta_{12}e^{\frac{-i\omega p_1 d}{c}} / (1+\mathfrak{R}_{01}\mathfrak{R}_{12}e^{\frac{-2i\omega p_1 d}{c}}) \quad (25)$$

where

$$\vartheta_{01} = 1 + \mathfrak{R}_{01} \quad (26)$$

$$\vartheta_{12} = 1 + \mathfrak{R}_{12}. \quad (27)$$

The formulae (22) and (25) are in agreement with the results already obtained.[4]

Reflection Method

As stated above, the difficulties of the determination of dielectric constant and loss factor

(4) J. A. Stratton, "Electromagnetic Theory", 1st Ed., 1941, p. 510.

by free wave method lie partly in the calculation of their values from the reflection or transmission coefficient experimentally obtained. In the following we shall report a convenient method for the determination of dielectric constant and loss factor from the reflection coefficient of a dielectric sheet placed on a metal plate. In the case where the refractive index is much greater than absorption coefficient, we can derive approximate formulae which enable us to compute these quantities from the experimental values of the reflection coefficient. In other cases when these approximate formulae do not hold, we may make tables which are convenient for the evaluation of these values.

1. Reflection of Electromagnetic Wave by a Dielectric Layer on a Metal.—The plane electromagnetic wave is considered to be linearly polarized and to be propagated from the air into a dielectric sheet on a metal at normal incidence as shown in Fig. 2. The complex reflection coefficient is to be expressed as follows:

$$\Re = \frac{\Re_{12}+\Re_{23}e^{-\frac{2i\omega pd}{c}}}{1+\Re_{12}\Re_{23}e^{-\frac{2i\omega pd}{c}}}. \qquad (28)$$

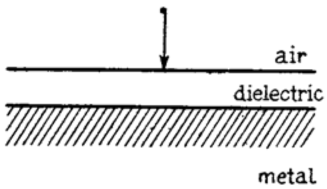

Fig. 2.

The equation (28) is essentially the same as the Eq. (22), but we have changed the subscripts in accordance with the convention adopted in other literatures.

Let ε', and ε'' respectively be the dielectric constant and loss factor of the dielectric and p^2 be $\varepsilon' - i\varepsilon''$. We have then

$$\Re_{12} = -\frac{(p-1)}{(p+1)}; \quad \Re_{23} = -1$$

and therefore,

$$\Re = -\left[\frac{-\Re_{12}+e^{-\frac{i2\omega pd}{c}}}{1+\Re_{12}e^{-\frac{i2\omega pd}{c}}}\right]. \qquad (29)$$

If we put

$$\Re_{12} = -R_{12}e^{-i\gamma}, \qquad (30)$$

then

$$R_{12} = \sqrt{\frac{(n-1)^2+k^2}{(n+1)^2+k^2}} \qquad (31)$$

and

$$\tan\gamma = \frac{2k}{(n^2-1)+k^2}. \qquad (32)$$

The square of the absolute value of $\Re$ is expressed as follows:

$$R^2 = \frac{\sinh^2\left(\frac{2\pi kd}{\lambda}+\frac{1}{2}\ln R_{12}\right)+\cos^2\left(\frac{2\pi nd}{\lambda}-\frac{\gamma}{2}\right)}{\sinh^2\left(\frac{2\pi kd}{\lambda}-\frac{1}{2}\ln R_{12}\right)+\cos^2\left(\frac{2\pi nd}{\lambda}+\frac{\gamma}{2}\right)}$$
$$= \frac{R^2{}_{12}+e^{-\frac{8\pi kd}{\lambda}}+2R_{12}e^{-\frac{4\pi kd}{\lambda}}\cos\left(\frac{4\pi nd}{\lambda}-\gamma\right)}{1+R_{12}^2e^{-\frac{8\pi kd}{\lambda}}+2R_{12}e^{-\frac{4\pi kd}{\lambda}}\cos\left(\frac{4\pi nd}{\lambda}+\gamma\right)} \qquad (33)$$

where λ denotes the wave length of the electromagnetic wave in the air. R^2 may be called reflection coefficient, and represents the ratio of the intensity of the reflected wave to that of the incident wave.

n is the refractive index and k is the absorption coefficeient of the material composing the dielectric sheet of the thickness d. They are related to p, ε', and ε'' through the Eqs. (5) (6a) and (6b).

If we determine the value of R^2 for various values of d, we can calculate n and k by Eq. (33). However, as R^2 is a transcendental function of n and k, it is difficult to obtain their values directly from the experimental values of R^2. Therefore, we must either find approximate formulae appropriate to the calculation, or make tables useful to the computation.

2. The Case of $n \gg k$.—If $n \gg k$, approximate formulae can be obtained rather easily. In this case we have

$$\gamma \doteqdot 0 \qquad (34)$$

$$R_{12} \doteqdot \frac{n-1}{n+1}. \qquad (35)$$

In Eq. (33), variable d appears in the form of nd or kd. Moreover nd appears only in the trigonometric functions and kd in the exponential functions.

According to the assumption $n \gg k$, the exponential functions in Eq. (33) change so slowly when compared to the trignometric functions that the former may be considered as constant in the small region of the variable d. Hence Eq. (33) may be rewritten as follows:

$$R^2 = \frac{a+b\cos\left(\frac{4\pi nd}{\lambda}\right)}{c+b\cos\left(\frac{4\pi nd}{\lambda}\right)} \tag{36}$$

where

$$a = R^2_{12}+e^{-\frac{8\pi kd}{\lambda}}$$

$$b = 2R_{12}e^{-\frac{4\pi kd}{\lambda}}$$

$$c = 1 + R_{12}{}^2e^{-\frac{8\pi kd}{\lambda}}.$$

a, b, c may be regarded as constant in the small region of variable d.

We denote the maximum and minimum values of R^2 as $R^2{}_M$ and $R^2{}_m$, and the corresponding values of d as d_M and d_m, respectively. They may be obtained by ordinal method of differentiation, and we find

$$R^2{}_M = \left(\frac{R_{12} + e^{-\frac{4\pi kd_M}{\lambda}}}{1+R_{12}e^{-\frac{4\pi kd_M}{\lambda}}}\right)^2 \tag{37}$$

$$R^2{}_m = \left(\frac{R_{12} - e^{-\frac{4\pi kd_m}{\lambda}}}{1-R_{12}e^{-\frac{4\pi kd_m}{\lambda}}}\right)^2 \tag{38}$$

$$d_M = \frac{2N\lambda}{4n} \tag{39}$$

$$d_m = \frac{(2N+1)\lambda}{4n} \tag{40}$$

where $N=0, 1, 2, \ldots.$

As stated above, our problem is to determine n and k (or ε' and ε'') from the experimental curve of R^2 *vs.* d. For this purpose we first determine n from the value of d_M or d_m experimentally obtained by Eq. (39) or (40). Next we calculate R_{12} by Eq. (35) using the value of n thus obtained. Then we determine k from the values of R_M or R_m by Eq. (37) or (38), the values of n, R_{12} and d_M (or d_m). Eq. (37) or (38) is a linear equation of exp $(-4\pi kd_M/\lambda)$ or exp $(-4\pi kd_m/\lambda)$, so we can obtain these values easily, from which we can determine the value k, using the tables of exponential function.(5) For higher approximation, we substitute the value of k into Eq. (31), and obtain the value of R_{12}. Using the value of R_{12} thus obtained, we can determine the value of k from Eq. (37) or (38) more exactly.

In the case when we can find many values of extrema though the value of k is very small we can determine the value of k from the decreasing rate of the extrema. Eq. (37) or (38) may be expanded with respect to $y = 4\pi kd/\lambda$, putting $e^{-y} = 1 - y + 1/2\,y^2 \cdots$, and as first order expansion, we have

$$R_M \doteqdot 1 - \frac{1-R_{12}}{1+R_{12}}y \tag{41}$$

$$R_m \doteqdot 1 - \frac{1+R_{12}}{1-R_{12}}y. \tag{42}$$

Differentiating with respect to $d/\lambda = z$, we have

$$\frac{dR_M}{dz} = -\frac{(1-R_{12})}{(1+R_{12})}4\pi k \tag{43}$$

$$\frac{dR_m}{dz} = -\frac{(1+R_{12})}{(1-R_{12})}4\pi k. \tag{44}$$

Thus from the curve R_M *vs.* d_M, or R_m *vs.* d_m, we can obtain the value of k.

3. The Case when the Assumption $n \gg k$ does not Hold.—In the case when $n \gg k$ does not hold, we can obtain the values of ε' and ε'' (or n and k) from the experimental values of R^2 according to the following method. We draw a curve which represents R^2 *versus* d/λ according to Eq. (33) for fixed values of ε' and ε'' (or n and k).

We draw another curve (which represents R^2 *vs.* d/λ according to Eq. (33)), for another fixed value of ε'' and the same value of ε' (or another fixed value of k and the same value of n). Thus we can obtain many curves representing R^2 *vs.* d/λ, a part of which is related to constant values of ε' and ε'', and the other to the same value of ε' and the different values of ε''. Then we have many curves of similar type which correspond to other fixed values of ε'. Thus we have many curves R^2 *vs.* d/λ corresponding to ε' and ε'' (or n and k) in the required region.

A curve R^2 *vs.* d/λ is characterized by two parameters ε' and ε''. Maximum value of R^2, $R^2{}_M$ or minimum value R^2, $R^2{}_m$ are determined principally by ε'', and the corresponding value of d, d_M and d_m by ε'.

From a curve, R^2 *vs.* d/λ, thus obtained, we mark $R^2{}_M$, $R^2{}_m$, d_M and d_m, and for a fixed value of ε' we draw curves $R^2{}_M$ *vs.* ε'' and

(5) K. Hayashi, "Kôtô Kansû Hyô" (Tables of higher functions), 1st Ed., 1945; "En oyobi Sôkyokusen Kansu Hyô" (Tables of circular and hyperbolic functions), 1st Ed., 1946.

R^2_m *vs.* ε''. Similarly, for a fixed value of ε'', we draw curves d_M *vs.* ε' and d_m *vs.* ε'.

To obtain the values of ε' and ε'' from the experimental values of reflection coefficients, we draw a curve R^2 *vs.* d/λ, and compare it with the curves calculated by the afore-described procedure. Thus we can obtain rough values of ε' and ε''. Using the value of ε'' thus obtained, we can obtain ε' from the curve d_M/λ *vs.* ε' or d_m/λ *vs.* ε' by interpolation. (The value ε' thus obtained is almost a true value, for the curves d_M/λ *vs.* ε' or d_m/λ *vs.* ε' are insensitive to the value ε''.) Similarly we can determine ε'' from the curve R^2_M *vs.* ε'' or R^2_m *vs.* ε'', using rough value of ε'. (The curves R^2_M *vs.* ε'' or R^2_m *vs.* ε'' are insensitive to the value ε'.) The above procedure is essentially the same when we use n and k instead of ε' and ε''.

Transmission Method

In the following a method will be reported, which consists in determining dielectric constant ε' and loss factor ε'' of a material from the relation of the transmission coefficient and thickness of dielectric sheet concerned.

1. The Formula of Transmission Coefficient.—The plane electromagnetic wave of unit amplitude is considered to be linearly polarized and to be propagated in the air. A dielectric sheet with plane boundary surface is placed as shown in Fig. 3.

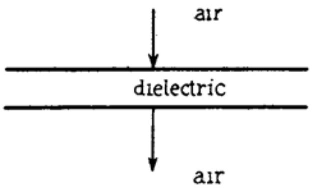

Fig. 3.

The direction of the propagation of incident wave is considered to be normal to the boundary surface. A part of the wave will be transmitted through the sheet characterized by dielectric constant ε', loss factor ε'' and thickness d. The complex amplitude of the electric field vector of the transmitted wave is given by Eq. (25). If we change the notation ϑ_{2}, and p_1, to ϑ, and p, and considering

$$\vartheta_{01}\vartheta_{12}=\frac{4p}{(1+p)^2},\quad \Re_{01}\Re_{12}=\left(\frac{p-1}{p+1}\right)^2$$

then we have

$$\vartheta=\frac{e^{-\frac{i2\pi pd}{\lambda}}\times\dfrac{4p}{(1+p)^2}}{1+\left(\dfrac{p-1}{p+1}\right)^2 e^{-\frac{i4\pi pd}{\lambda}}}. \tag{45}$$

If we write

$$\frac{p-1}{p+1}=-R_{12}e^{-i\gamma}$$

then ϑ is expressed as follows, considering $p=n-ik$.

$$\vartheta=e^{-\frac{2\pi kd}{\lambda}}\times\frac{e^{-\frac{i2\pi nd}{\lambda}}(1-R_{12}{}^2e^{-i2\gamma})}{1-R_{12}{}^2e^{-i2\left(\frac{2\pi nd}{\lambda}+\gamma\right)}\cdot e^{-\frac{4\pi kd}{\lambda}}}$$

The transmission coefficeint of the sheet D^2 may be defined as $D^2=|\vartheta|^2$, and expressed as:

$$D^2=e^{-\frac{4\pi kd}{\lambda}}\times\frac{(1+R_{12}{}^4)-2R_{12}{}^2\cos 2\gamma}{1+R_{12}{}^4e^{-\frac{8\pi kd}{\lambda}}-2R_{12}{}^2e^{-\frac{4\pi kd}{\lambda}}\cos\left(\dfrac{4\pi nd}{\lambda}+2\gamma\right)}$$
$$=\frac{\sinh^2(-\ln R_{12})+\sin\gamma}{\sinh^2\left(\dfrac{2\pi kd}{\lambda}-\ln R_{12}\right)+\sin^2\left(\dfrac{2\pi nd}{\lambda}+\gamma\right)} \tag{46}$$

2. Approximate Formulae.—(i) If $R_{12}{}^2\ll 1$, we have

$$D^2 \doteqdot e^{-\frac{4\pi kd}{\lambda}} \tag{47}$$

In this case, the value of k can be obtained easily from the curve D^2 *vs.* d/λ experimentally obtained.

(ii) If $n\gg\kappa$, we have the relations expressed with Eqs. (34) and (35). Eq. (46) is rewritten as

$$D^2\doteqdot e^{-4\pi kx}\times\frac{(1-R_{12}{}^2)^2}{1+R_{12}{}^4e^{-8\pi kx}-2R_{12}{}^2e^{-4\pi kx}\times\cos 4\pi nx} \tag{48}$$

where $x=d/\lambda$.

As stated in the case of reflection method, exponential function in Eq. (48) change very slowly when compared to trigonometric function according to the assumption that $n\gg k$, so that the former can be considered as a constant in a small region of variable x. Eq. (48). can be rewritten as follows, considering $\exp(-4\pi kx)=a$, a constant, in this region.

$$D^2=\frac{a(1-R_{12}^2)^2}{1-2aR_{12}^2\cos 4\pi nx+R_{12}^4a^2} \quad (49)$$

The maximum and minimum value of D^2 and corresponding values of d, can at once be obtained. If we denote these values D_M^2, D_m^2, and the corresponding values of d, d_M and d_m, then we have

$$D_M=e^{-\frac{2\pi k d_M}{\lambda}}\times\frac{(1-R_{12}^2)}{(1-e^{-\frac{4\pi k d_M}{\lambda}}R_{12}^2)} \quad (50)$$

$$D_m=e^{-\frac{2\pi k d_m}{\lambda}}\times\frac{(1-R_{12}^2)}{(1+e^{-\frac{4\pi k d_m}{\lambda}}R_{12}^2)} \quad (51)$$

$$d_M=\frac{2N}{4n} \quad (52)$$

$$d_m=\frac{(2N+1)}{4n} \quad (53)$$

$N=0, 1, 2, \ldots.$

To obtain the values of n and k (accordingly ε' and ε'') from the curve D^2 *vs.* d/λ which was experimentally obtained, we find at first the value of n from Eq. (52) or (53), then the value of k from Eq. (50) or (51), calculating the value of R_{12} from Eq. (35). Using these value of n and k thus obtained, more accurate value of k can be obtained from Eq. (50) or (51), calculating the value of R_{12} from Eq. (31).

(iii) If $k\ll 1$, the value of k may be obtained from the curves of extrema *vs.* x. Omitting the detailed calculations, we can finally express the derivatives of D_M and D_m with respect to x as follows:

$$\frac{dD_M}{dx}\doteqdot-\left(1+\frac{2R_{12}^2}{1-R_{12}^2}\right)\cdot 2\pi k \quad (54)$$

$$\frac{dD_m}{dx}\doteqdot-\left(\frac{1-R_{12}^2}{1+R_{12}^2}\right)\frac{(1-R_{12})^2}{(1+R_{12}^2)}\cdot 2\pi k \quad (55)$$

3. The Case when the Assumption $n\gg k$ is not valid.—In this case the values of ε' and ε'' can be obtained from the curve of D^2 *vs.* d/λ which was experimentally obtained by a similar method as described in the case of reflection method. That is, we draw many curves of D^2 *vs.* d/λ for the values of ε' and ε'' in the suitable region, and obtain the values of ε' and ε'' by interpolation.

Summary

The reflection and transmission coefficient of electromagnetic waves by multiple layer has been obtained theoretically.

A convenient method has been deviced to determine dielectric constant and loss factor of a material in a form of a sheet placed on a metal, from the reflection coefficeint.

A convenient method has also been developed to determine the dielectric constant and loss factor from experimental curve of transmission coefficient versus thickness of dielectric sheet concerned.

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